



The NASA, Van Allen Probe spacecraft orbit Earth so high up that there is hardly any air at all. Scientists use the term 'density' to measure how many kilograms of gas there are in each cubic-meter of space, but when the density is too low, a unit like kg/m^3 is not very helpful. That's because instruments often measure individual atoms, and kg/m^3 is just too big a unit! It's like using 'kilometers' to measure the size of a bacterium.

A much more convenient unit is 'atoms/ m^3 '. This tells scientists immediately just how often their very sensitive instruments will be affected by their environment.

Problem 1 – The density of the Van Allen belts is typically about 900 atoms/m^3 . How many atoms would you expect to find in a box that measures 15 centimeters on a side?

Problem 2 – The opening to one of the Van Allen Probes spacecraft instruments is about 10 cm^2 . As the satellite completes one orbit, it travels about 70,000 km. How many atoms will pass through the spacecraft instrument window each orbit?

Problem 3 – How many kilometers would the spacecraft have to travel in order to encounter 9 million atoms?

Problem 1 – The density of the Van Allen belts is typically about 900 atoms/m^3 . How many atoms would you expect to find in a box that measures 15 centimeters on a side?

Answer: $10 \text{ cm} = 0.15 \text{ meters}$, so the volume of the box is $0.15 \times 0.15 \times 0.15 = 0.0034 \text{ meters}^3$. Then the number of atoms is density \times volume = $900 \times 0.0034 = \mathbf{3 \text{ atoms}}$.

Problem 2 – The opening to one of the Van Allen Probes spacecraft instruments is about 10 cm^2 . As the satellite completes one orbit, it travels about 70,000 km. How many atoms will pass through the spacecraft instrument window each orbit?

Answer: Convert the area into square meters, and the orbit length into meters to get

$$\begin{aligned} \text{Area} &= 10 \text{ cm}^2 \times (1 \text{ m}/100\text{cm}) \times (1\text{m}/100\text{cm}) \\ &= 0.001 \text{ m}^2, \end{aligned}$$

and $70,000 \text{ km} \times (1000 \text{ m}/1 \text{ km}) = 70,000,000 \text{ m}$.

Then volume = area \times length to get $(0.001 \text{ m}^2) \times (70,000,000 \text{ m}) = 70,000 \text{ m}^3$. Now multiply this 'swept out' volume by the density to get the number of atoms that passed through the window: $900 \text{ atoms/m}^3 \times 70,000 \text{ m}^3 = \mathbf{63 \text{ million atoms}}$.

Problem 3 – How many kilometers would the spacecraft have to travel in order to encounter 9 million atoms?

Answer: The window area is 0.001 m^2 and the density of atoms is 900 atoms/m^3 .

You want 9 million atoms, so

$$9 \text{ million} = 900 \times \text{area} \times \text{length}$$

$$9 \text{ million} = 900 \text{ atoms/m}^3 \times 0.001 \text{ m}^2 \times \text{Length}$$

Length = $9 \text{ million} / 0.9 = 10,000,000 \text{ meters!}$ **This equals 10,000 kilometers.**